Butterworth phase characteristics and the stopband specification

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As the story goes, the phase lag of a low-pass filter—in this case a Butterworth filter—increases with its order. While true under some assumptions, the statement actually lacks one or two preconditions: First of all the actual frequency domain where the phase is compared, but also the bandwidth of each filter. The statement is certainly true in either of these cases:

- At very high frequencies
- For any frequency; if the same bandwidth is specified for each filter order

Neither is of much value when designing a low-pass filter, however. In that case, the typical approach would be to prescribe a certain minimum attenuation D_s above some frequency ω_s. At the same time, if phase matters, it is probably considered below some frequency specific to the application.

What happens if you compare filters adhering to the same stopband specification?

The diagrams in Figs. 1 through 9 below display several Bode plots. Within each figure, the plots are subject to the same stopband specification. This is clearly reflected in the magnitude plots of each figure. The phase plots suggest that it is possible, in each case, to single out the one filter featuring the smallest lag—at least up to the point where the phase drops below approximately $-45°$. This limit increases, and approaches $-90°$, for attenuation values above 40dB. Thus, for minimizing the phase lag in the lower end of the spectrum, the best filter order n for each desired attenuation D_s is shown in this remarkably regular and easily memorized table:

D_s[dB]	10	20	30	40	50	60	70	80	90
n	2	3	4	5	6	7	8	9	10

This item accompanies the paper

Dessen, F.: Optimizing Order to Minimize Low-Pass Filter Lag. Circuits, Syst. and Signal Process. (2018).

and is identified therein as 'Online Resource 1'. Some amount of information is duplicated in order to render both sufficiently self contained.

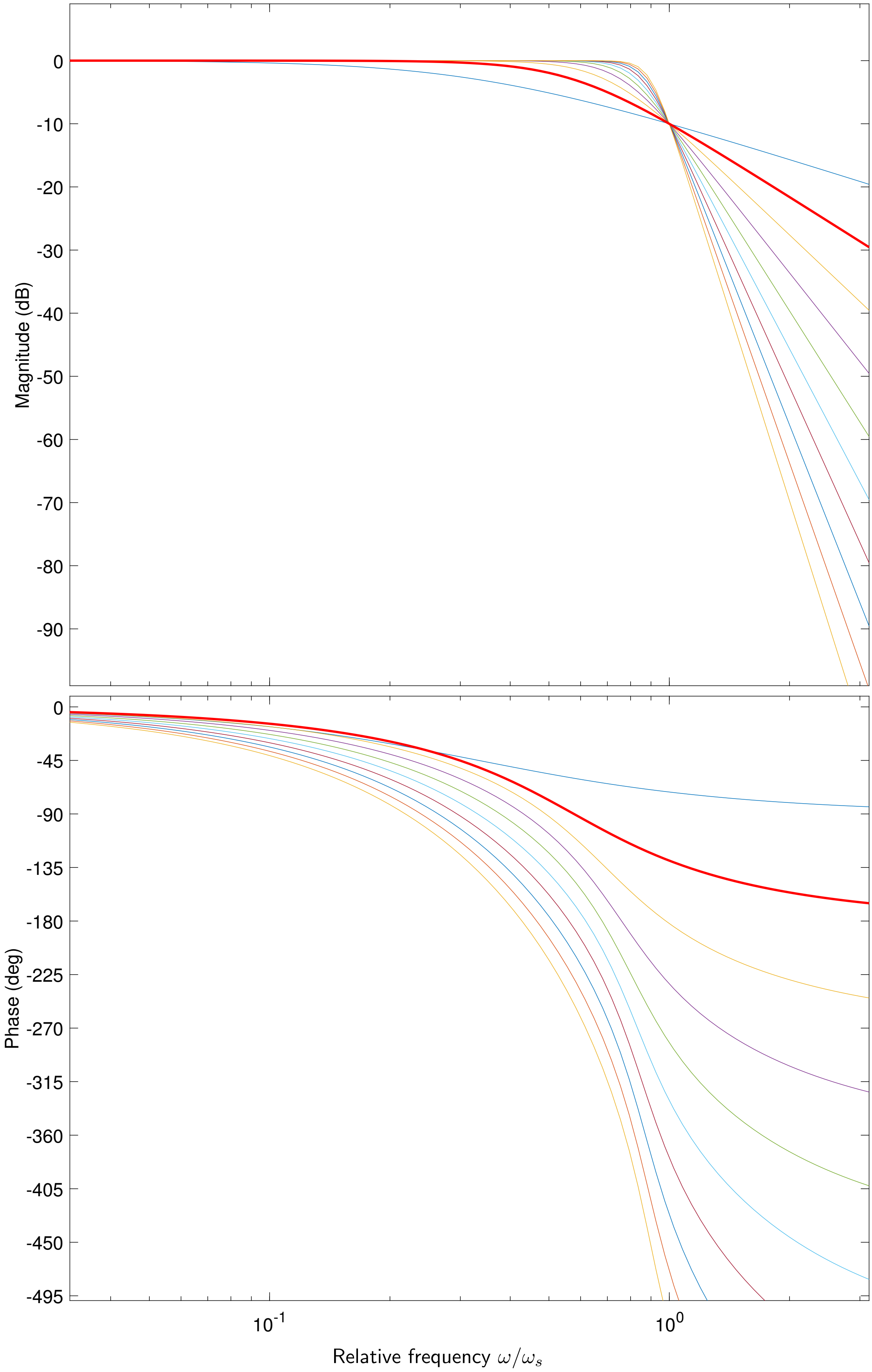

Fig. 1 Butterworth filters of orders 1 through 10. All filters have the same attenuation of 10 dB at $\omega = \omega_s$. Order 2 is highlighted, since it features the smallest phase drop.

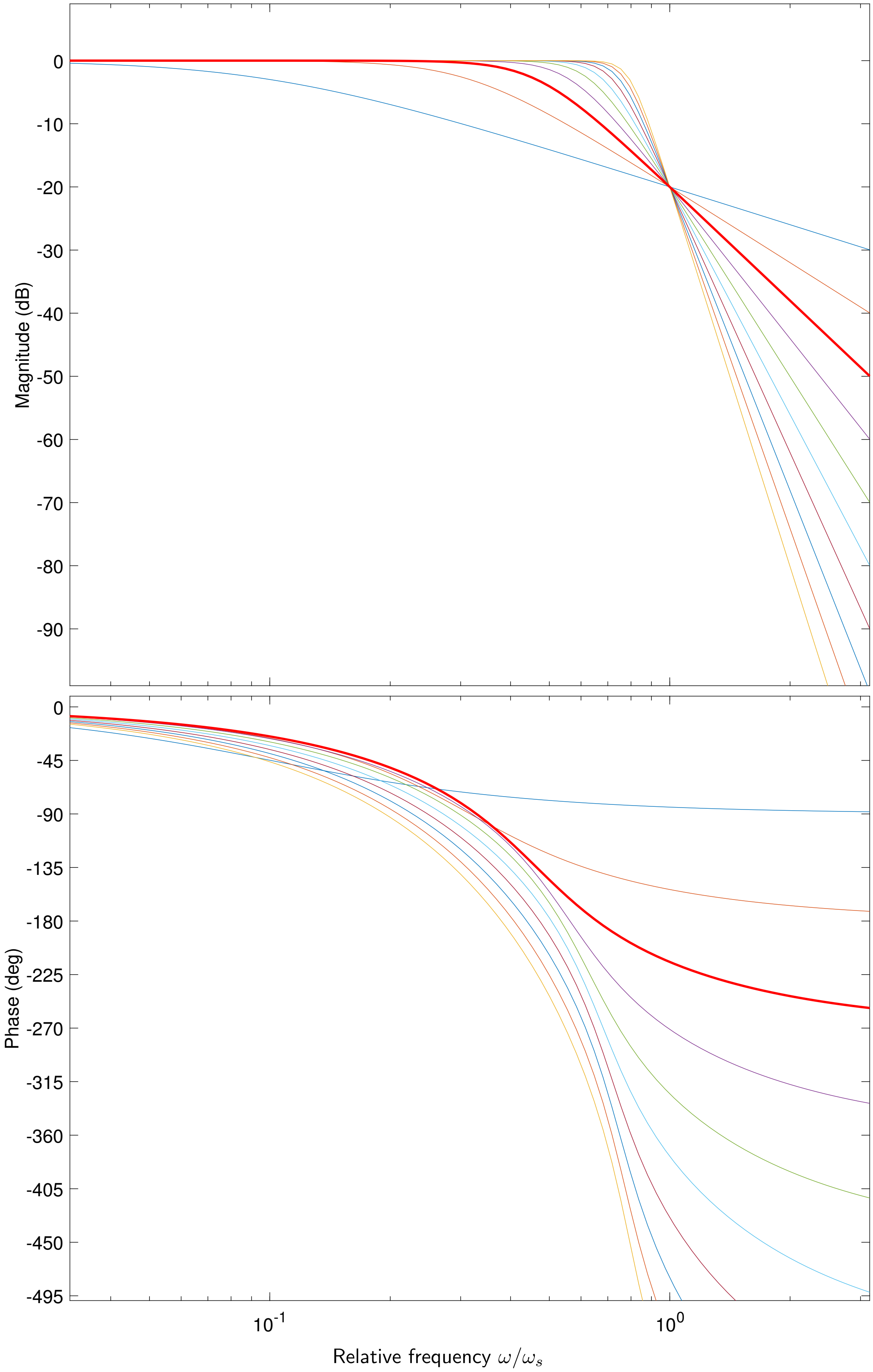

Fig. 2 Butterworth filters of orders 1 through 10. All filters have the same attenuation of 20 dB at $\omega = \omega_s$. Order 3 is highlighted, since it features the smallest phase drop.

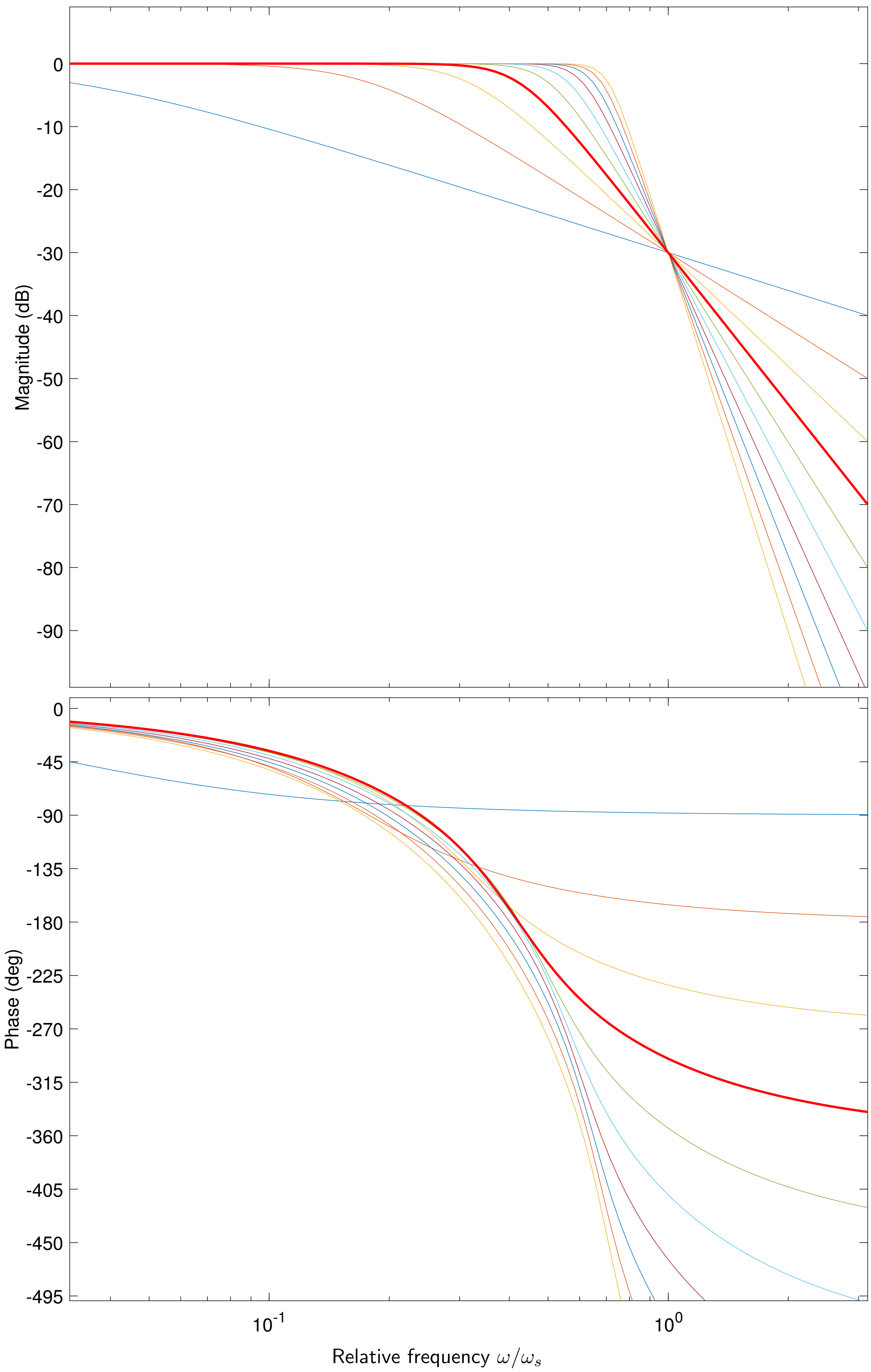

Fig. 3 Butterworth filters of orders 1 through 10. All filters have the same attenuation of 30 dB at $\omega = \omega_s$. Order 4 is highlighted, since it features the smallest phase drop.

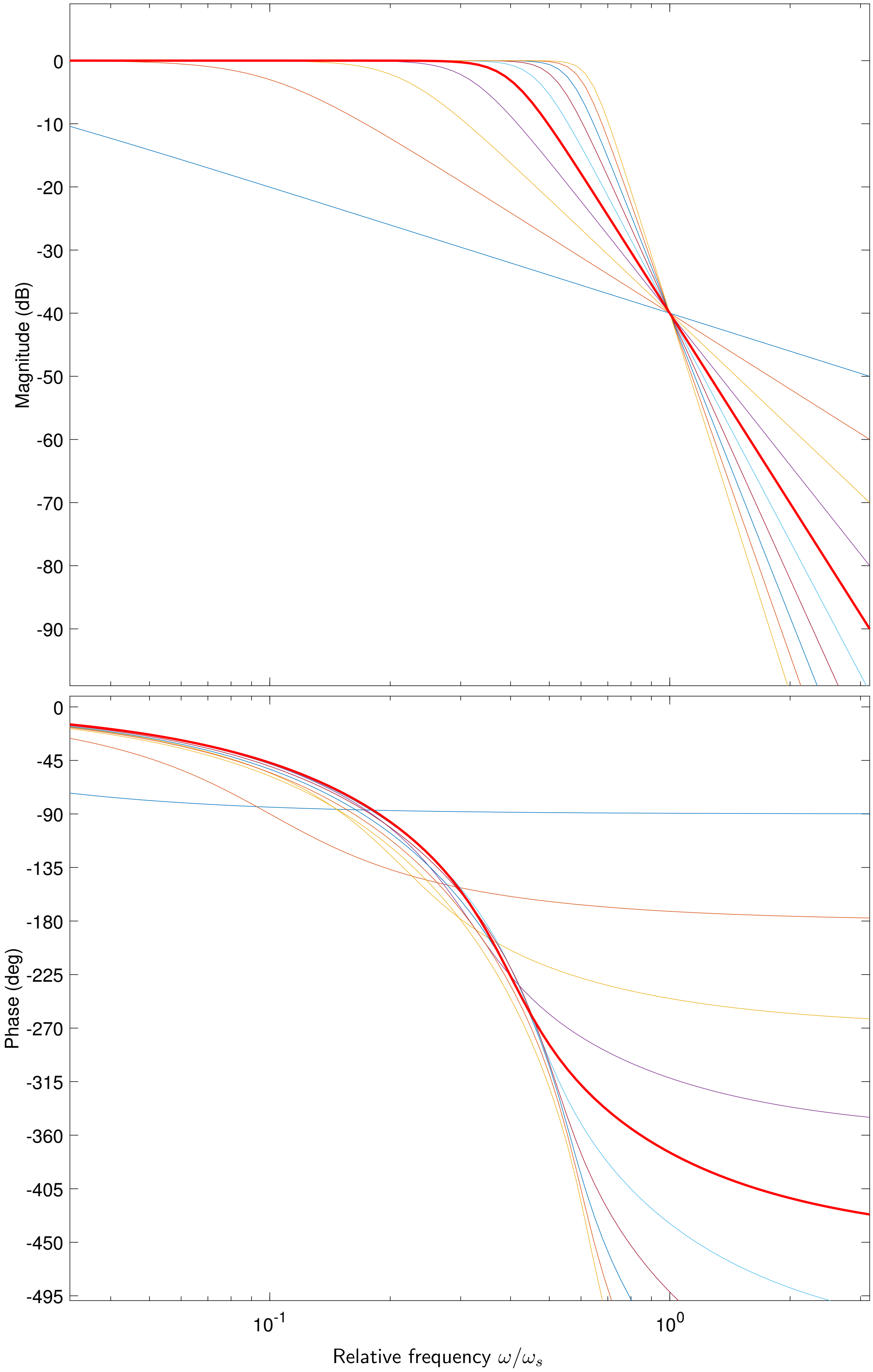

Fig. 4 Butterworth filters of orders 1 through 10. All filters have the same attenuation of 40 dB at $\omega = \omega_s$. Order 5 is highlighted, since it features the smallest phase drop.

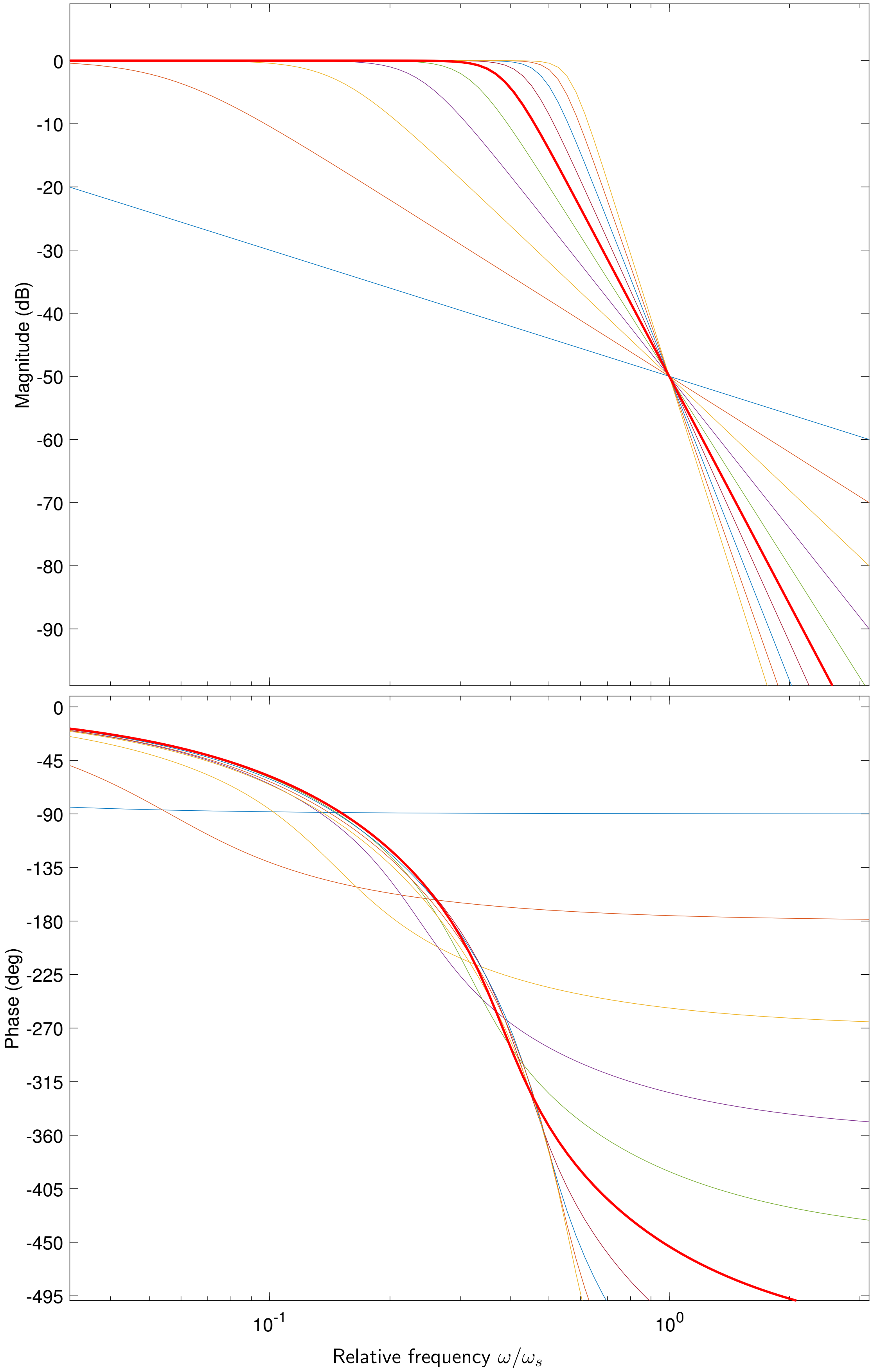

Fig. 5 Butterworth filters of orders 1 through 10. All filters have the same attenuation of 50 dB at $\omega = \omega_s$. Order 6 is highlighted, since it features the smallest phase drop.

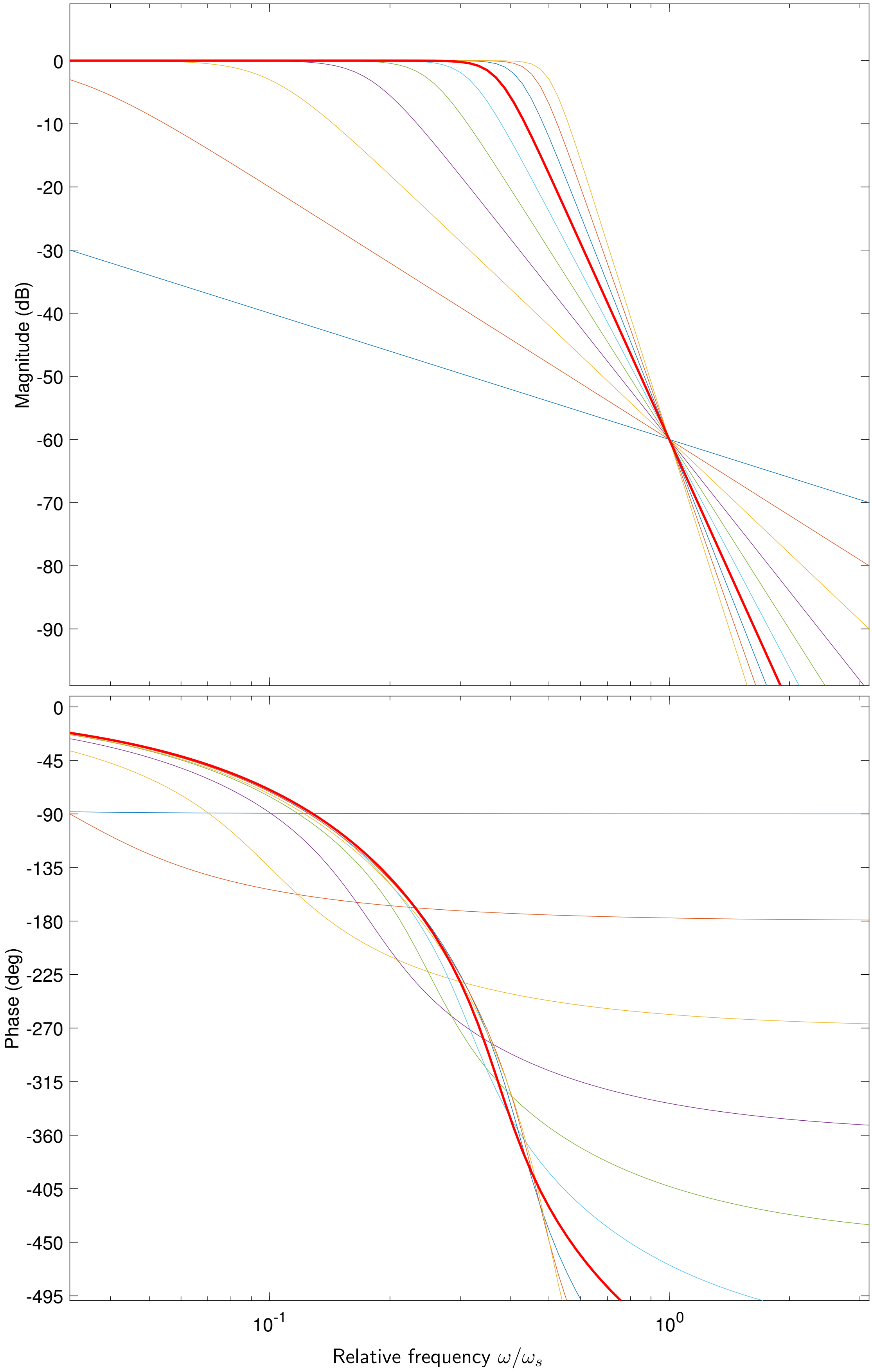

Fig. 6 Butterworth filters of orders 1 through 10. All filters have the same attenuation of 60 dB at $\omega = \omega_s$. Order 7 is highlighted, since it features the smallest phase drop.

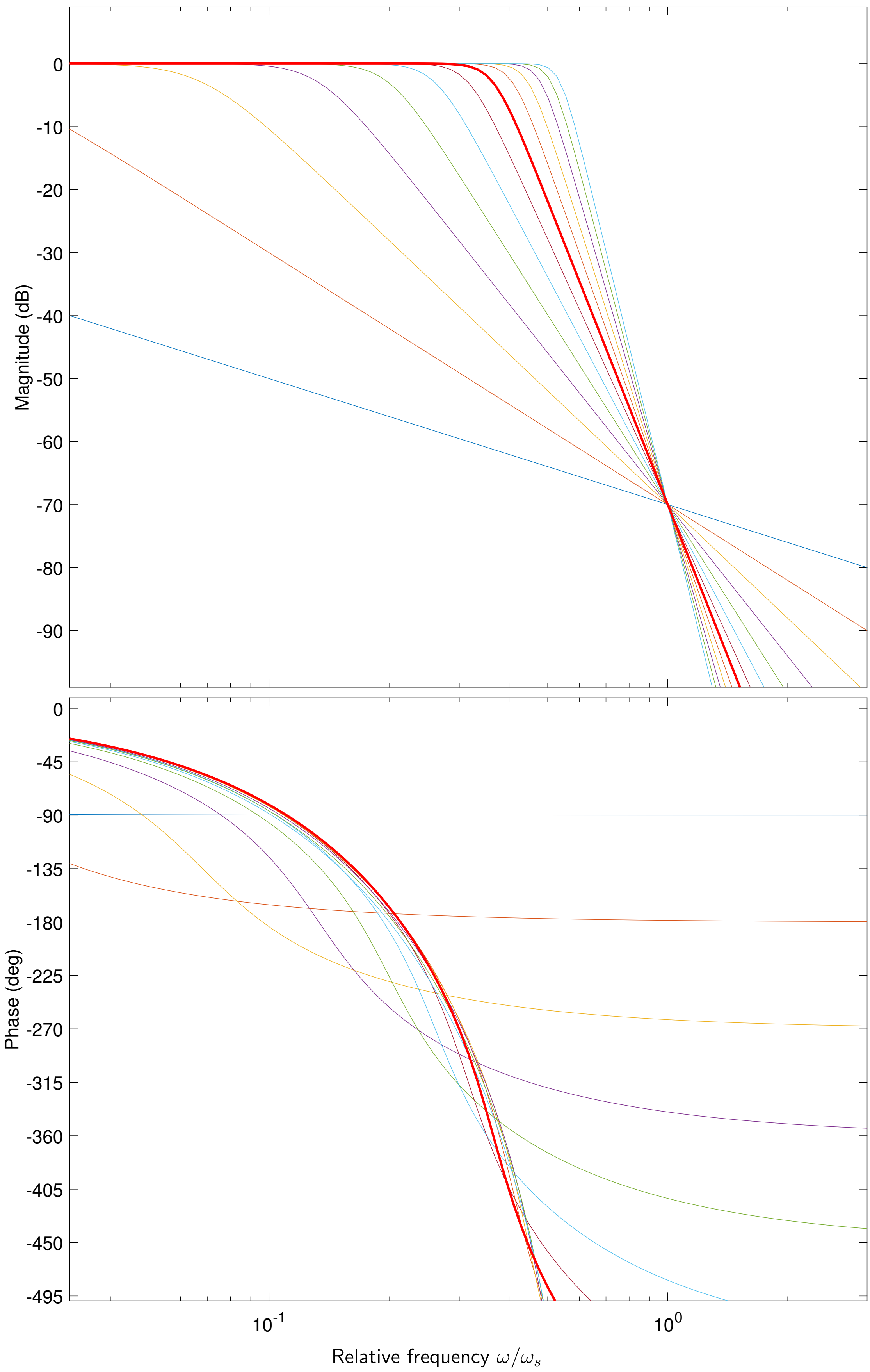

Fig. 7 Butterworth filters of orders 1 through 13. All filters have the same attenuation of 70 dB at $\omega = \omega_s$. Order 8 is highlighted, since it features the smallest phase drop.

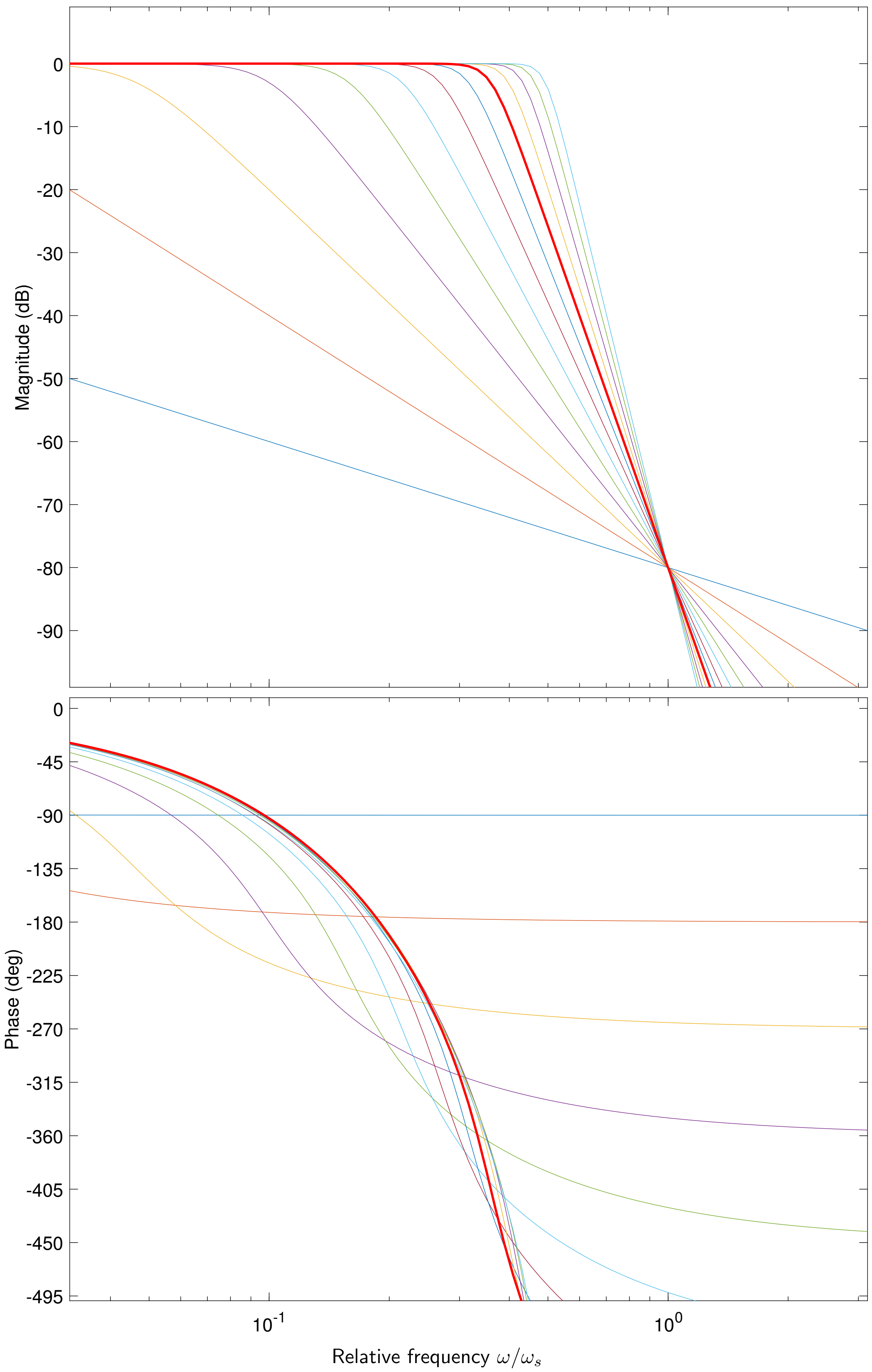

Fig. 8 Butterworth filters of orders 1 through 13. All filters have the same attenuation of 80 dB at $\omega = \omega_s$. Order 9 is highlighted, since it features the smallest phase drop.

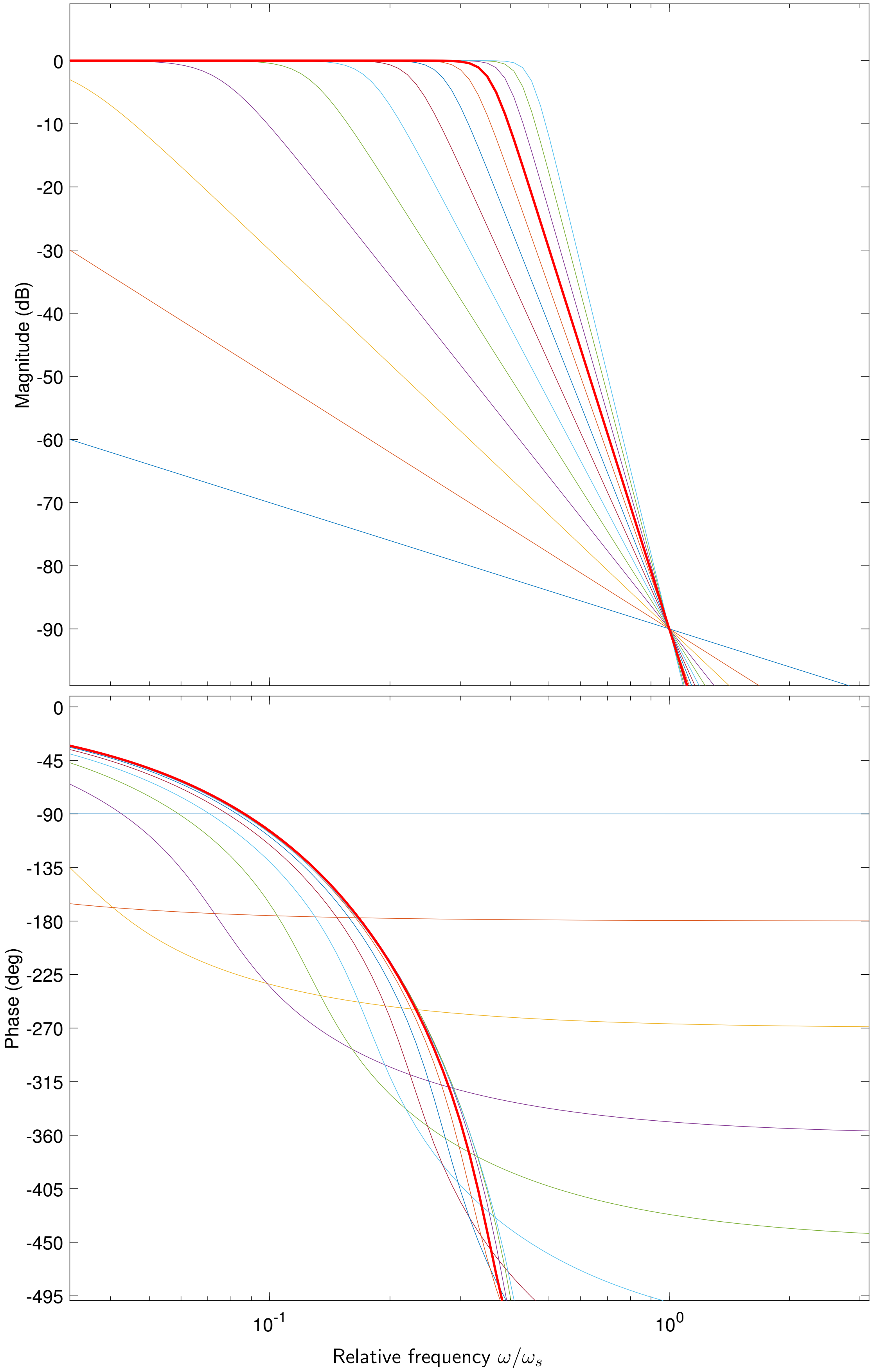

Fig. 9 Butterworth filters of orders 1 through 13. All filters have the same attenuation of 90 dB at $\omega = \omega_s$. Order 10 is highlighted, since it features the smallest phase drop.